Cycle lengths in graphs of given minimum degree

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Abstract

In a graph, k cycles are *admissible* if their lengths form an arithmetic progression with common difference one or two. Let G be a 2-connected graph with minimum degree at least $k \geq 4$. We prove that

(1) G contains k admissible cycles, unless $G \cong K_{k+1}$ or $K_{k,n-k}$;

(2) G contains cycles of lengths ℓ modulo k for all even ℓ, unless $G \cong K_{k+1}$ or $K_{k,n-k}$;

(3) G contains cycles of lengths ℓ modulo k for all ℓ, unless $G \cong K_{k+1}$ or G is bipartite.

In addition, we show that if k is even and G is 2-connected with minimum degree at least $k-1$ and order at least $k+2$, then G contains cycles of lengths ℓ modulo k for all even ℓ. These findings provide a stability analysis of the main results on cycle lengths in graphs of given minimum degree in [J. Gao, Q. Huo, C. Liu, J. Ma, A unified proof of conjectures on cycle lengths in graphs, International Mathematics Research Notices 2022 (10) (2022) 7615–7653]. As a corollary, we determine the maximum number of edges in a graph that does not contain a cycle of length 0 modulo k for all odd k.

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