RESEARCH RESULTS OF THE HUN-REN ALFRÉD RÉNYI INSTITUTE OF MATHEMATICS IN 2024

Research in algebra

A new quantity called triangulant was introduced in linear algebra. It is a number that is assigned to two matrices as a polynomial in the matrix entries, and it is zero if and only if the two matrices have a left, resp. a right eigenvector that are perpendicular to each other. The triangulant was applied to provide a new extremal characterization of mutually unbiased bases, which play an important role in quantum information theory. As a generalization of the triangulant, higher-order triangulants were also introduced. The k-th triangulant of two n-by-n matrices is zero if and only if the two matrices have nontrivially intersecting invariant subspaces of dimension k and n-k, respectively.

They proved the existence of the maximum entropy random walk for i.i.d. weights on non-amenable groups in the low-temperature phase. They investigated the maximum speed of diffusion processes on the universal cover of compact surfaces with negative curvature. They studied the bundling phenomenon in hardcore, positive-density ergodic line processes, providing an effective entropy estimate for its discrete lattice analog. They further examined the existence of growth on unimodular random trees and the Anderson localization phenomenon for flow-invariant measures. Regarding the Poisson boundary problem, using the Lubetzky-Sly theorem, they showed that on a random d-regular graph, the limit of diffusion-weighted sampling is either the boundary of the d-regular tree or collapses to a point; that is, there is no intermediate phase transition.

The directed Cayley diameter of a finite abelian group was expressed in terms of the sizes of the cyclic direct summands of the group. An algorithm was developed that provides lower and upper bounds for the directed Cayley diameter of any non-abelian finite group. The algorithm has been implemented in the GAP computer algebra system.

Research in artificial intelligence

In the context of research on the theoretical foundations of artificial intelligence, an optimal transport-based generative model from the family of Wasserstein Autoencoder models was defined. In this approach, the positions of hidden points are optimized over the entire training data set. Several metrics have been implemented to evaluate these models, and it has been shown for certain prior distributions that global optimal transport improves the performance of the generative models compared to minibatch-based models.

The elements of the NTK matrix can be approximated more accurately by inverse cosine distances between corresponding activations as the network depth increases. This observation also allows a more accurate estimation of spectral properties. Their results show that the parameters of the activation function essentially determine how fast the condition number of the NTK matrix stabilizes with increasing depth. An absolute value based activation function is preferable in this respect compared to, for example, ReLU.

The TrOCR optical character recognition model has been updated and optimized at the Historical Archives of the State Security Services. The TrOCR model was retrained on the data set of the Hungarian National Archives. The Retrieval-Augmented Generation framework was further developed in two directions: along the standard vector-based retrieval solutions and the so-called GraphRAG approach based on graph-based representation. In addition, they have been working on a solution for automatic filtering of sensitive information using the in-context learning capabilities of large language models.

They tried to solve a Helly-type problem using machine search methods. The computer analysis revealed a connection with quadratic programming, which eventually led to the discovery of a "paper and pencil" proof that did not use computer methods.

It was seen that if the normalized form of a Hadamard matrix contains three elements of -1, then the matrix belongs to one of two families of concrete Hadamard matrices.

Algorithms have been given for the resolution theorems of quadratic Fourier analysis: decomposition of functions interpreted on Abel groups into structured and quasi-random parts, and determination of quadratic harmonic components.

A higher-order generalization of the notion of groups was introduced and developed, involving the theory of so-called nilspaces. With these new structures, it may be possible to describe the behaviour of Gowers norms on noncommutative groups.

Research in probability theory

Several dynamic random graph models were investigated. In the degree-constrained network model, they understood how the degree constraint delays the appearance of the giant cluster. Two high profile physics papers have been published: one modelling the operation of quantum communication networks, the other modelling the construction of physical networks (e.g. brain, vascular, river networks).

A random tree model in a random environment is introduced, interpolating between the Uniform and Minimal Spanning Trees of a graph. The phase transitions of its diameter on the complete graph are determined.

At the interface of statistical physics and information theory, they investigated the question of whether one can guess the value of a function from a small random subset of the input spins. For the Ising model of magnetization, they found that at the critical temperature the total magnetization can be guessed, but at higher temperatures no function can be.

One of the main open questions in the theory of infinite group actions on probability spaces and Borel equivalence relations is whether Borel amenability implies hyper finiteness. Two relaxations of this conjecture have been proved.

In a large class of epidemic Markov chains, the accuracy of the deterministic NIMFA approximation of the epidemic curve was determined.

Invariance principles (convergence to Brownian motion) have been proved for two- and three-dimensional Lorentz gases in physically relevant cases.

They proved functional central limit theorems for telecommunication (queuing) systems when service times are not independent but merely stationary. The previously known convergence rates to the equilibrium state have also been improved.

They have developed a method for finding confidence bounds for Markov chains in random environments that is applicable even if the environment fails stationarity (but satisfies some other regularity properties). These results were obtained via estimating the strong mixing coefficients of the respective processes.

They showed how much the mixing speed of an asymmetric random walk on the circle can be accelerated by adding random edges.

A method has been worked out to infer total variation convergence from weaker convergence concepts of probability measures (Fortet-Mourier, Wasserstein), under additional moment and smoothness hypotheses. Comparison between the respective convergence rates has also been established. Applications to stochastic dynamical systems with memory and to Malliavin calculus have been developed.

Research in analysis

Members of the Analysis department have achieved results in research areas in accordance with the preliminary plans.

In Fourier analysis, concerning the Fuglede-problem, they have constructed "lonely weak tiles" which are neither proper tiles, nor spectral sets. Also, using the Delsarte LP-method in cyclic groups, they have constructed functional tilings which violate the T2 condition of Coven and Meyerowitz. For Delsarte-type extremal problems they have established the existence of an extremal function after appropriate modification of the problem.

In quantum-information theory they have characterized triplets of MUBs in terms of Hadamard cubes.

In the theory of quantum optimal transport, they have given a partial proof (and also some numerical evidence) of the conjecture about metric properties of optimal transport distances defined by quantum channels. They have established a general, non-quadratic version of the optimal transport problem based on quantum channels, and using this general scheme they have investigated fundamental properties of p-Waaserstein distances and divergences. They have characterized the isometries of p-Wasserstein spaces (for all positive values of p) for the max-norm on the plane and on the unit square.

In approximation theory, they have introduced a new general notion of the "weakly analytically connected" property for sets, and proved that this property implies the validity of the Bernstein-Markov property for arbitrary almost everywhere positive weights. They have successfully applied this notion to prove subexponential growth of the derivatives of multivariate polynomials.

In mathematical physics, they have approximated the spectrum of linear operators: for self-adjoint, trace-class operators, they could converge to the spectrum in Hausdorff metric, and they could calculate the necessary moments in practice for a large class of physically relevant operators.

As an application of analysis in number theory, they proved new density results about the roots of Beurling-zeta functions.

Research in number theory

In the theory of Dirichlet L-functions and the Riemann zeta function an important role is played by the so-called density theorems. One of the most famous problems is the so-called density hypothesis that can be formulated for more general functions too, like the Dedekind zeta functions, the Beurling zeta functions and functions of the Selberg class. They improved known density theorems for Dedekind zeta functions. They proved a general zero density theorem on the Selberg class of functions. The result verifies the Density Hypothesis in the strip when the real part of the variable is at least 0.9, under the assumption that the degree of the function does not exceed 2. In the other cases they obtained a density theorem weaker than the Density Hypothesis. They proved a new zero-density estimate for Beurling zeta functions.

The sup-norm problem for Hecke-Maass cusp forms on the group $SU(n,n-1)$ was considered. They have seen earlier that in the case n=2 the corresponding matrix counting problem is not solvable. In the case n=3 they could describe the matrices in question explicitly by using methods of algebraic number theory, and consequently answer the original question in the same negative manner as for n=2. However, the argument proves an earlier unsolved question to characterize those 6-dimensional integral vectors which can be extended to an icube of maximal ran.

Let G be the modular group. If z and w are two given points in the upper half plane and R is a real number tending to infinity, the hyperbolic circle problem asks for the asymptotic estimation of the number of points equivalent to z under G and lying in the non-Euclidean circle around w of radius R. The error term of the best pointwise estimate known so far is $\exp(2R/3)$. It was known that taking $z=w$ and averaging the error term over the points of a fundamental domain this estimate can be improved. They succeeded to prove that taking the local square mean of the error term the estimate can be also improved. The estimate given for the local square mean is exp(cR), where c is any constant satisfying $c>9/14$.

An almost cover of a discrete set of points is a collection of hyperplanes that covers all points except one. According to the classical Alon-Füredi theorem, every almost cover of the vertex set of a d-dimensional hypercube consists of at least d hyperplanes. Using the polynomial method known from combinatorial number theory they succeeded in proving a generalization of this theorem.

Research of extremal combinatorial structures

Two finite families of convex sets in R^3 are intersecting if every blue set intersects every red one. They proved that if all blue sets lie in parallel planes and all red sets lie in parallel planes, then there is a line intersecting either every blue set or every red set.

Connection of generalized Turán problems and degree based topological indices was investigated. Topological indices are used in chemical graph theory to estimate certain chemical properties of molecules. Methods from the theory of generalized Turán numbers were used in the study of topological indices, reproving and extending earlier results.

They gave abstract generalizations of the Erdős-Stone-Simonovits theorem. Erdős-Ko-Rado range hypergraph Turán problems, rainbow Turán problems in directed graphs and excluded set system problems were studied. They proved an Erdős-Ko-Rado type theorem for subgraphs of perfect matchings. Stability theorems for (0,1)-configurations were investigated.

A monotone Boolean function of n variables has k reasons to be 1, if its disjunctive normal form has k members. They determined the minimum number of elementary tests needed to determine the function in both the adaptive and non-adaptive cases.

They proved that the lexicographically last m sets of k-element subsets of an n-element set maximize the volume of the intersecting pairs, where the size of the intersection is used to measure its volume.

They finished their work on easy testability of monotone classes of posets. They finished a paper on treeings, where the measurable Hall theorem fails, solving multiple questions and conjectures and submitted it to Inventiones. Their constructive methods can be applied to control the space of circulations in a graphing. They developed a method to construct a graphing for every natural number n such that the space of circulations has dimension exactly n. Their 50 pages long paper on a measurable Hall's theorem for hyperfinite graphings has received positive reports from Duke.

They continued their work on expressing subclasses of NP via graph colorings and homomorphisms.

Strongly possible integrity constraints of relational databases were examined from extremal and algorithmic point of view, furthermore minimal size and structure of Armstrong tables of cardinality constraints were investigated that lead to interesting graph theory questions.

Problems in graph theory

They found a counterexample to a 25-year-old conjecture called the Merino-Welsh conjecture for matroids. This is a conjecture concerning an inequality between some evaluations of the Tutte polynomial. The conjecture has two forms, one for graphs and one for matroids. The latter is the more general version, and they showed that that one is not true.

They proved results about the minimum number of edges required for a graph in a special metric space the distances of any two points of which within the graph is at most twice or at most just a little more than twice their distance within the metric space. In particular, they proved a super-linear lower bound on the number of edges needed in the first case and a linear upper bound in the second case.

They gave upper bounds in terms of the maximum degree on the minimum number of colors needed for coloring the edges of a planar, bipartite planar or outerplanar graph in such a way that the edges of all 4-cycles receive 4 distinct colors.

They verified a conjecture of Hakimi and Schmeichel about the maximal number of pentagons in an n-vertex planar graph. They also characterized the extremal graphs.

They proved an optimal version of the so-called Quantitative Fractional Helly Theorem that states that if a given fraction of (d+1)-tuples of n convex sets in the d-dimensional Euclidean space have an intersection of volume at least 1, then some positive fractions of the n sets have an intersection with some volume one can prescribe as a function of the dimension only.

They showed that the complementary Lovász theta number behaves similarly to the fractional chromatic number with respect to the Mycielski construction in the sense that its value for a graph determines its value on the Mycielskian of the same graph. A formula giving the value of the latter in terms of the former was also found.

They refuted a 20-year-old conjecture of Pach and Tardos concerning the maximum number of 1's in certain 0-1 matrices, namely those that do not contain the bipartite incidence matrix of some forest.

Combinatorics and network theory

They have continued the research undertaken in the DYNASNET project. Their most important results were the construction of the convergence concept of monotone submodular functions and the construction of the limit object. The theory fits nicely with the theory of graphings: circular matroids of locally-globally convergent graph series tend to the matroid of the limit graphing. They have also investigated possible variants of matroid multiplication, and extended the well-known notion of "coupling" from probability theory to set functions, including matroids via rank functions.

Concerning physical networks: researchers of the department explored the impact of volume exclusion on the local assembly of linear physical networks, where nodes and links are hard-core rigid objects. They introduced a minimal 3D model and showed that the kinetics of link adhesion is logarithmic, as opposed to the algebraic growth in lower dimensions, and they attribute this qualitatively different behavior to a spontaneous delay of depletion forces caused by the 3D nature of the problem. Their findings offered a benchmark to study the local assembly of physical networks, with implications for non-equilibrium nest-like packings.

Concerning the research of mathematical modelling of epidemics: the final version of a paper on the mathematical analysis of the "switchover" phenomenon their colleagues have previously observed in epidemic spread was prepared. The metapopulation epidemic spread modelling framework, one of the flagship deliverables of the EGLAB National Laboratory project, has been further developed. This framework is ready to be integrated into the Respicast European Epidemic Alert System. A behavioural epidemic model has been developed, in which interactions between people adaptively change according to the epidemic situation and the level of individual awareness. They have developed a method to infer behavioural events (e.g. superpropagation events) based on mutations in the genetic sequences of the virus. They have developed a project on the effects of natural and random immunisation processes and their impact on herd immunity against subsequent outbreaks.

Research in mathematical methodology

Teaching mathematics through guided discovery lies in the focus of all research activities of the Department. The "This Is How We Would Teach It" project investigates how high school mathematics curricula can be taught more effectively, involving practicing mathematics teachers. The lesson plans developed in this project are publicly available on the group's website.

Members of the Department examine the practice of teaching mathematics through guided discovery. The group approaches this from several angles. On one hand, it develops educational materials aimed at high school students with a discovery-oriented mindset. On the other hand, it also focuses on how to support more teachers in adopting this approach. One outcome of this research is the publication: Juhász P, Szász R, Szűcs G, Varga E. (2024): Professional Support in Teaching Mathematics through Guided Discovery: The Role of Agency on Multiple Levels. Education Sciences, 14(7), 769. — published in a Q2-ranked journal.

In addition, the department develops lesson plans and supplementary materials for teachers, enabling them to step beyond rigid curricula and highlight the usefulness, real-world relevance, and inherent beauty of mathematics to their students.

An analysis has also been completed of a 2023 pilot project on the teaching of statistics in high schools. The results have been submitted for publication in the article titled Advancing Statistical Inference: A Comprehensive Exploration of Curriculum Development, Teacher Seminars, and Teacher Perceptions in Hungarian Secondary Schools.

Introducing Hungary's approach to mathematical talent development to the international research community is also a key aspect of the group's work. The group is engaged in two international collaborations. One is a longstanding cooperation with the University of Montpellier, the other is a joint research effort with the Freudenthal Institute, focusing on realistic mathematics education.

The department is also involved in community-building efforts for Hungarian mathematics teachers, aiming to support and enhance their professional work. Staff members play a major role in organizing the professional program of the Rátz László Annual Conference — the largest yearly gathering of Hungarian mathematics teachers. Furthermore, the department provides opportunities for practicing teachers to participate in mathematics education research and to deepen their understanding of guided discovery teaching through workshops.

Research in algebraic geometry and differential topology

They studied the lattice cohomology of curve singularities, and they proved several structure theorems for them. They established the definition and general properties of a very general lattice cohomology theory. They searched for invariants of curve singularities embedded into surface singularities. They proved that the topological type of a sandwiched singularity admits only a finite number of Milnor fibers provided by the smoothing of the analytic structures.

They proved the existence of an infinite family of 4-torsion classes in the integer coefficient cohomology of oriented Grassmannians. They characterized a family of 6-dimensional complex Hadamard matrices. They conjectured that the integer coefficient cohomology of $OGr_{k(n)}$ oriented Grassmannians does not contain 4-torsion for k=3,4. They verified this conjecture for k=3. They proved that the Segre-Stiefel-Whitney class of a stratified space pulls back to the Segre-Stiefel-Whitney class of the preimage, if the map is sufficiently transversal.

For certain 4-dimensional manifolds they studied the existence of exotic structures using embedded links. They answered a question of Kronheimer, proving that there are 2-component links in S^4 that are not split by an S^3, but that are split by 3-manifolds with the same homology as S^3. They proved a relation between the signatures of an arborescent 2-component link and the d-invariants of its double branched covers.

They completed a paper regarding the exotic embedding of the five-fold connected sum of the real projective plane to the standards sphere. They continued the examination of exotic smooth structures on four-manifolds with fundamental group Z_2 and got an almost complete answer in the non-spin case. The results generalize to some other fundamental groups as well.

They proved a conjecture of Liebeck, Nikolov, and Shalev conjectured for the structure of finite simple groups in some key cases. They investigated the Floer theory of complex 2-dimensional Hitchin moduli spaces and the filtration induced on their second cohomology. They observed that the degree of a class is determined by the multiplicity of the given component of the non-reduced fiber.

Members of the group have presented at more than 25 international conferences.

Research in convex and combinatorial geometry

They characterized exponential valuations defined on lattice polygons in the plane in both basic cases – if the valuations are valued either at measurable functions or at analytic functions. In the latter case, when the valuations are valued at analytic functions, the structure of the space resembles the structure of the space of modular forms. In the case of $SU(m)$ equivariant tensor valued valuations, they determined the dimensions of the ones with given ranks, and the weights of the corresponding $U(m)$ actions.

For determining the optimal density of packings of equal spheres in dimensions 8 and 24 using the theory of modular forms, Maryna Viazovska has received the Fields medal. The HUN-REN researchers managed to prove the stability of this result combining analytic and geometric tools.

In every dimension they constructed Cech complexes with Betti numbers whose order of magnitude is maximal. In particular, they gave n unit balls in Euclidean 3-space that enclose roughly n^3 "holes".

They also improved the best-known bounds for the number of graphs that can be obtained as the intersection graph of n x-monotone curves, any pair of which share at most 1 point. The upper bound uses an almost 20-year-old result of some members of the Department of Geometry according to which the Vapnik-Chervonenkis dimension of these graphs is bounded.

A set is bi-Sidon, if their pairwise sums are all different, and the pairwise products are also all different. They proved that every n-element set of reals has a bi-Sidon subset whose size is much larger than $n^{1/3}$, the best previously known estimate.

A set of n points in the plane is called dense, if the ratio of the maximum and minimum distance determined by them is of order square root of n. For the special case of complete graphs $K(n)$, drawn by straight-line edges on a dense set of n vertices, they managed to prove the conjecture of Bose, Hurtado, Rivera-Campo, and Wood, according to which $K(n)$ splits into cn plane graphs, for some $c>0$.

They studied how many comparability subgraphs are needed to partition or cover the edge set of a perfect graph, and showed that for almost all perfect graphs two are enough. On the other hand, they showed that in some cases an arbitrarily large number of comparability subgraphs might be necessary.

Suppose that we have two crossing-free embeddings of a graph, with the same vertex positions and we wish to reconfigure the first embedding to the second one by rerouting one edge at a time, through crossing-free embeddings. They proved that it is always possible for forests on the torus.

Research in set theory

They investigated the properties that are between countable compactness and pseudocompactness. They have succeeded in constructing examples that separate these properties in a number of cases, some provable in ZFC and others only consistently.

They continued studying the cardinal sequence properties of scattered spaces in two directions. Building on their earlier work, they explored locally compact scattered spaces, seeking to identify the broadest class of cardinal sequences that can be proven to correspond to LCS spaces. They pursued a systematic study of Lindelöf *P*-spaces. They found that for LLSP spaces, no ZFC results comparable to those for LCS spaces can be expected; instead, even for spaces with finite Cantor-Bendixson height, independence results arise.

They introduced two topological games, which allowed them to define cardinal invariants related to the T_0-pseudoweight of spaces. They also proved theorems about these invariants. They have constructed several regular and sigma-compact topological spaces that do not have countable tightness but in which the closures of all discrete subspaces possess countable tightness. This solved a problem posed by V. Tkachuk.

They solved a problem of Erdős: Is there a planar set that is of linear measure 1 in every line? It has turned out that the answer is independent of the ZFC axioms.

They studied the dynamics of generic homeomorphisms of dendrites, focusing on the shadowing property. They achieved promising partial results. Continuing their earlier investigations, they obtained a complete description of all automorphism invariant measures defined on certain homogeneous structures.

The natural topology on the automorphism group of a countably categorical, countably stable structure can be reconstructed from the purely algebraic properties of the automorphism group. They generalized this result to countably categorical, stable structures.

The Artificial Intelligence Department proposed a common investigation on how it is possible to search formulas with "almost same meanings" in an automated (computer based) manner. Our approach is based on the following idea: two formulas have almost same meanings if the symmetric difference of their meanings has "small" measure with respect to some automorphism invariant measure mentioned above. Hence, in a fortunate scenario, our results on automorphism invariant measures may serve as a basis for applications in artificial intelligence developments.

Erdős Center
Organizing thematic semesters, Workshops and Summer Schools on various current topics in mathematics

In 2024, the Erdős Center hosted three thematics semesters, each had been selected earlier via an open call. These thematic semesters constitute the Milestones proposed for the year 2024.

1) Fourier Analysis and Additive problems (Spring, 2024)

Organizers: Ruzsa Imre (Rényi Institute), Oriol Serra (Barcelona Tech), Kiss Gergely (Rényi Institute), Matolcsi Máté (Rényi Institute), Somlai Gábor (Rényi Institute/ELTE)

This thematic semester hosted two major Workshops and one Summer School.

2) Measurable Combinatorics (Spring, 2024)

Organizers: Jan Grebik (Warwick), Alexander Kechris (Caltech), Oleg Pikhurko (Warwick), Stevo Todorcevic (Toronto), Vidnyánszky Zoltán (ELTE)

This thematic semester hosted one major Workshop and one Summer School, and the latter was part of the series „Young Set Theory Workshop” that is organized in every year in a different country over the World.

3) Fractals and Hyperbolic Dynamical Systems (Fall, 2024)

Organizers: Szász Domokos (BME), Bálint Péter (Rényi Institute/BME), Simon Károly (Rényi Institute/BME), Bárány Balázs (BME), Tóth Imre Péter (Rényi Institute/BME)

This thematic semester hosted two major Workshops and two Summer Schools.

In 2024, the Erdős Center applied for a grant supporting Summer Schools and postdoc scholarships supported by the Simons Foundation.

LIST OF IMPORTANT PUBLICATIONS IN 2024

- Stipsicz AI, Szabó Z: On the minimal genus problem in four-manifolds, In: Feehan PMN, Ng LL, Ozsvath PS (szerk.) Proceedings of Symposia in Pure Mathematics, American Mathematical Society (2024) pp. 215-232. 18 p.
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- Bárány I: A matrix version of the Steinitz lemma, JOURNAL FUR DIE REINE UND ANGEWANDTE MATHEMATIK 2024:809 pp. 261-267. 7 p. (2024)
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- Stipsicz AI, Szabó Z: Definite four-manifolds with exotic smooth structures, JOURNAL FUR DIE REINE UND ANGEWANDTE MATHEMATIK 2024:817 pp. 267-290. 24 p. (2024)
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OUTSTANDING ACHIEVEMENTS IN 2024

Unit distance avoiding point sets in the plane

What fraction of the plane can be colored so that no two-colored points are exactly one unit apart. According to a conjecture of Pál Erdős, this fraction cannot reach one quarter. Several research groups published partial results, gradually refining the initial density estimate of 0.2857 to 0.2544. According to the result of the researchers of the Rényi Institute, the density in question cannot exceed 0.247, thus proving the conjecture of Erdős. The results were published in the journal *Mathematical Programming*. The result was also reported by *Quanta Magazine*, an international standard for scientific dissemination.

The breakthrough came when researchers combined techniques of analysis, geometry and combinatorics to develop a common generalization of the earlier methods. With this approach, they reduced the problem to a search problem for a set of points with special properties. The properties required are too complex to be realistic to find the right set of points using paper and pencil, but using artificial intelligence they found a shape consisting of 23 points (Figure 1).

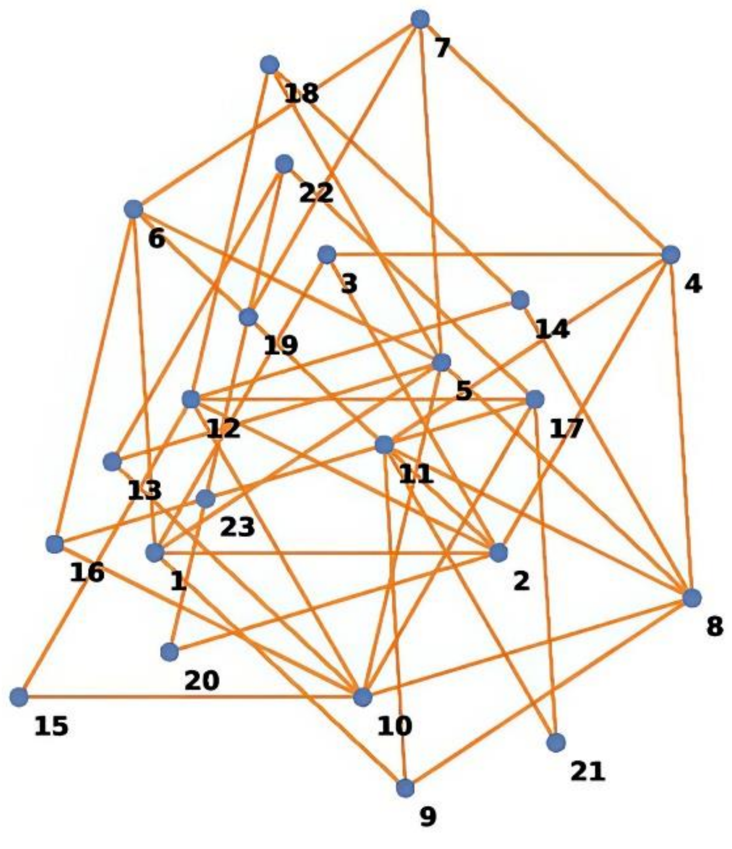

Figure 1: The 23-vertex graph

Densest ball packings

Lattice packings by equal balls have been intensively investigated since the 18th century using methods coming from number theory, harmonic analysis and discrete geometry. The study of non-lattice packings grew out of the former director of their institute, László Fejes Tóth's oeuvre. One of his celebrated results is that the so-called "hexagonal packing" has optimal density in the plane (Figure 2). Concerning the optimal density of packings of equal spheres in dimensions 8 and 24, Fields Medal laureate Maryna Viazovska proved that the optimal densities are attained by certain lattice arrangements. The HUN-REN researchers strengthened Maryna Viazovska's result by proving that if the density of a packing is close to be optimal, then the structure of the balls resemble the optimal lattices. The paper has been published by the 200 years old *Crelle's Journal*.

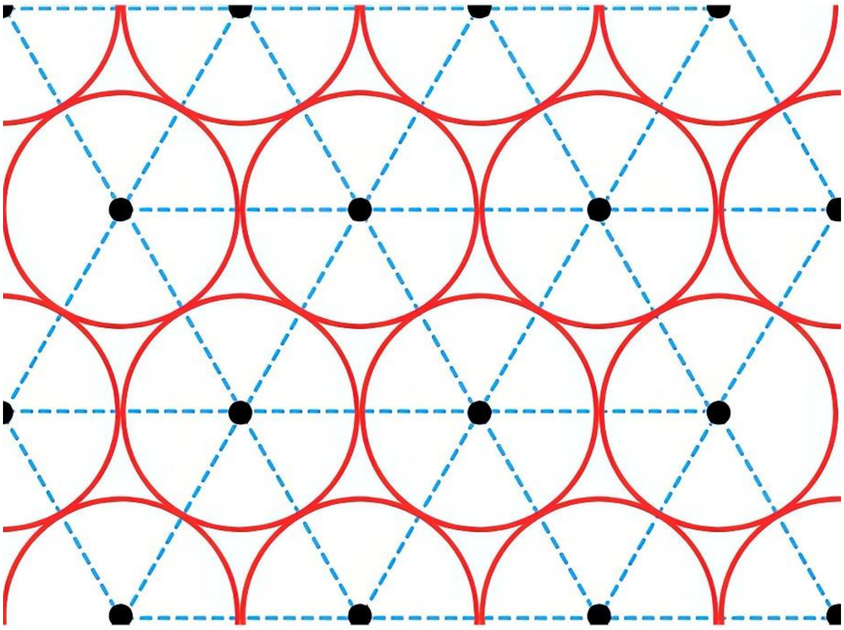

Figure 2: Hexagonal packing